Joint inversion for inner core rotation, inner core anisotropy, and mantle heterogeneity

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Abstract. Observational evidence for differential rotation of the inner core has recently been reported from time-dependent travel times of the seismic inner core phase, PKP(DF). These analyses are hindered by potential biases from the mantle heterogeneity and systematic earthquake location errors, which have raised doubts about the reality of the inferred differential rotation. Here we report time-dependent PKP(DF)-PKP(BC) differential travel times of a total of 611 measurements obtained from 92 earthquakes in the South Sandwich Islands (SSI) over the past nearly half a century recorded at the College, Alaska station (COL) and Alaska Seismic Network (ASN). The events are relocated using the joint hypocenter determination (JHD) technique [Dewey, 1971]. Travel time residuals at COL from SSI earthquakes over a period of 45 years and at three additional stations from SSI earthquakes over periods of about 22 years show steady increases with time; the temporal trends are demonstrated to be statistically very significant. With dense samples of the pathway at different time periods, we use a joint inversion technique that allows us to separate the time-dependent inner core structure from the time independent mantle biases. Surprisingly, the inner core rotation seems resolvable even from digital data observed at ASN in the 1990s (spanning over 8 years) alone in such a joint inversion. The results provide strong support for a differential inner core rotation. The inferred rotation rate ranges from 0.3° to 1.1° per year faster than the mantle when we take into account the uncertainty in the tilt of the anisotropy axis of the patch of the inner core that was sampled. A westward inner core rotation can be ruled out.

1. Introduction

The dynamo action in the metallic fluid outer core, which generates and maintains the Earth's magnetic field, was expected to drive the conducting inner core to rotate through electromagnetic coupling [Gubbins, 1981; Glatzmaier and Roberts, 1995]. Observational evidence for a differential rotation of the inner core has recently been reported from seismic waves that penetrate the inner core [Song and Richards, 1996; Su et al., 1996; Creager, 1997]. Song and Richards [1996] examined seismic waves that travel through the inner core from earthquakes at almost the same location but decades apart to the same monitoring station. To reduce biases from mantle heterogeneity and earthquake location errors, they used differential travel times between PKP(DF) and PKP(BC) phases, which go through the mantle and most of the outer core very closely together but the DF phase goes through the inner core and the BC phase turns at the bottom of the outer core. They found that the BC-DF differential times along certain pathways, including a pathway from earthquakes in South Sandwich Islands (SSI) to the station at College, Alaska (COL), have changed systematically with time. The temporal changes were interpreted as evidence for a differential inner core rotation, which moves the axis of the inner core's known anisotropy [Morelli et al., 1986; Woodhouse et al., 1986; Song, 1997], and the rotation rate was estimated to be about 1°/yr. Subsequent estimates of the rotation rate vary by orders of magnitude from 3°/yr by Su et al. [1996] to 0.2°-0.3°/yr or even as low as 0.05°/yr by Creager [1997]. An attempt by Souriau [1998] to detect the inner core rotation was not successful, although she could not rule out a rotation of less than 1°/yr. A recent study of normal mode data by Laske and Masters [1999] found that inner core differential rotation is essentially zero (to within the error of ±0.2° per year) over the last 20 years.

Two major issues underlying the uncertainty of the determination of the differential inner core rotation are (1) potential biases from the mantle heterogeneity and (2) possible systematic earthquake location errors.
These two issues together with other concerns (such as possible biases from heterogeneous event magnitudes) have led Souriau et al. [1997] and Souriau [1998a,b] to raise doubts on the claims that a differential inner core rotation has been detected, which we address more thoroughly in a separate paper [Song and Li, 2000]. In this study we report time-dependent BC-DF travel time observations with a total of 611 measurements from 92 SSI earthquakes, which have been relocated using the joint hypocenter determination (JHD) technique [Dewey, 1971], over the past nearly half a century to an array of 101 stations in Alaska. We show that because the mantle biases do not move with the inner core, they can be separated from time-dependent inner core structure in a joint inversion with such dense sampling. Our results provide strong support for a differential rotation of the inner core and put a tight constraint on the estimate of the rotation rate.

2. Time-Varying Differential Travel Time Observations

Following Song and Richards [1996], we first examine BC-DF differential times at a fixed observing station that has operated for a long time along a pathway that is sensitive to inner core anisotropy. A long recording history is critical in establishing the consistency of temporal changes and examining their robustness. The high-quality station COL started operating in 1935 with a horizontal component long-period instrument (0.1 Hz) and was equipped with a three-component short-period (1.0 Hz) Benlof instrument in 1949 [Glover, 1977]. It became a Worldwide Standardized Seismograph Network (WWSSN) station in 1964 and was upgraded to digital WWSSN recording in 1982 [Ganse and Hutt, 1982], and broadband digital instruments in 1987 and in 1991. We found original paper records of SSI earthquakes at COL between 1951 and 1966 (only vertical components are useful for the PKP waves of our interest) in a warehouse of the U.S. Geological Survey office in Golden, Colorado, extending the previous measurements at COL by Song and Richards [1996] further back 15 years.

Also in Alaska, the Geophysical Institute of the University of Alaska at Fairbanks (UAFGI) has been operating the Alaskan Seismic Network (ASN) with over 100 stations since the late 1960s. Most ASN stations are short-period vertical component instruments with dominant gain at 1 Hz [Glover, 1977]. Virtually complete archives of seismograms are still available at UAFGI, but most of the archives are in microchip form (developed) for which the use of waveforms is impossible. Paper seismograms (helicorders) are available for a limited number of stations; the scale on the short period paper records is 60 mm between minute marks. Digital recordings of analog signals started around 1989 with a sampling rate of 120 samples per second. Of the paper records obtained, stations at Gilmore Dome (GLM, very close to COL), Yukon (FYU), McKinley (MCK), and Sheep Creek Mountain (SCM) have the most complete continuous recordings.

Figure 1 summarizes the results obtained from records at COL, FYU, MCK, and SCM. The original paper records are scanned into digital images with a resolution of 600 by 600 dots per inch. The paper records are then digitized from these scanned images. All of the differential travel times at COL are measured by cross correlating DF and BC waveforms from either the digitized analog seismograms or the digitally recorded ones. The paper seismograms at ASN stations (including FYU, MCK, and SCM), however, were recorded on a drum with a pen of fixed arm length, resulting in arced traces. Because of the difficulty of such traces, the BC-DF times from the paper records at FYU, MCK, and SCM are measured by hand as follows. We first measure the distance between the middle of a DF pulse and the middle of the corresponding BC pulse along the zero-amplitude baseline. We repeat the measurement for an adjacent DF pulse and its corresponding BC pulse. We then average the two measurements to obtain the BC-DF time (scaled by the distance between minute marks nearby).

The resolution of the hand measurement is one pixel (one dot) or 0.042 s. For the digital records available in the 1990s at FYU, MCK, and SCM, we use both the cross-correlation technique and the hand measurement method. The difference between the BC-DF times obtained from 31 records at FYU, MCK, and SCM using the hand measurement method and the BC-DF times obtained using the cross-correlation technique is 0.008 ± 0.038 (1σ), that is, not significant. The BC-DF residuals from the cross-correlation measurements show slightly smaller scatter at each of the three stations, suggesting the cross-correlation technique is probably superior. To be consistent in processing the data of different time periods, we use hand measurements for FYU, MCK, and SCM in Figure 1.

We observe clear time dependence of the BC-DF times at all four stations we examined (Figure 1). The new measurements at COL for earthquakes in the 1950s and early 1960s are consistent with the temporal change observed previously [Song and Richards, 1996]. The trend of gradual increase in the BC-DF times at COL is striking; over the past 45 year period (1951 through 1995), the BC-DF times have increased by 0.54 s. Similar temporal changes were also observed at FYU, MCK, and SCM (which are 140 to 340 km apart from each other and from COL) from SSI events from about 1950 to 1998. The rates of changes are compatible with that of COL. Table 1 summarizes the results of the linear regressions of the observed travel time residuals on the event occurrence times:

\[ v = v_0 + b(T - T_0) \]

where \( v \) is the observed velocity perturbation (the observed residual normalized by the time that the ray path spend in the inner core), \( T \) is the event occurrence time, \( T_0 = 1998.0 \) is the reference time, the fitted inter-
cept $v_0$ gives the velocity perturbation at time $t_0$, and
the fitted slope $b$ gives the observed velocity change per
unit time. Assuming that all the measurements have
the same standard deviation, the standard deviation of
the intercept $v_0$, and that of the slope $b$, are also given
in Table 1.

To assess our observations that the $BC$-$DF$ residuals change with time, we test the null hypothesis that
the slope $b$ is zero [e.g., Freund and Wilson, 1997]. Using
Student's t test, the level of significance at which
the null hypothesis is unacceptable (meaning the prob-
ability that a slope departs from zero more than $sb$

Table 1. Results of Linear Regressions of Observed Velocity Perturbations on Time

<table>
<thead>
<tr>
<th>Station</th>
<th>$N$</th>
<th>Time Span, yr</th>
<th>Distance Span, deg</th>
<th>$\tau^{1C}$, s</th>
<th>$\langle \theta, \phi \rangle$, (deg,deg)</th>
<th>$v_0$, %</th>
<th>$\sigma_{v_0}$, %</th>
<th>$b$, %/yr</th>
<th>$\sigma_b$, %/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>COL</td>
<td>54</td>
<td>1951.89-1995.38</td>
<td>150.2-152.3</td>
<td>128.8</td>
<td>(8.4, 74.2)</td>
<td>2.563</td>
<td>0.030</td>
<td>0.00925</td>
<td>0.00117</td>
</tr>
<tr>
<td>FYU</td>
<td>15</td>
<td>1974.48-1995.01</td>
<td>149.5-151.1</td>
<td>124.3</td>
<td>(9.7, -70.3)</td>
<td>2.019</td>
<td>0.033</td>
<td>0.01046</td>
<td>0.00226</td>
</tr>
<tr>
<td>MCK</td>
<td>26</td>
<td>1974.48-1998.10</td>
<td>149.9-152.0</td>
<td>128.8</td>
<td>(7.6, -76.5)</td>
<td>2.468</td>
<td>0.031</td>
<td>0.01150</td>
<td>0.00249</td>
</tr>
<tr>
<td>SCM</td>
<td>14</td>
<td>1975.10-1998.10</td>
<td>148.9-150.4</td>
<td>121.3</td>
<td>(9.6, -9.3)</td>
<td>2.410</td>
<td>0.028</td>
<td>0.01481</td>
<td>0.00197</td>
</tr>
</tbody>
</table>

$N$ is the number of observations, $\tau^{1C}$ is the average of the times the rays travel through the inner core; $\theta$ and $\phi$ are the averaged latitude and averaged longitude of the ray turning points in the inner core; and $v_0$, $b$, $\sigma_{v_0}$, and $\sigma_b$ are the estimated linear regression parameters and their standard deviations described in the text.
by chance) is the probability that the variable \( |t| \) is greater than or equal to the critical (actual) value

\[
|t_c| = \left| b/\sigma_b - r \sqrt{(N-2)/(1-r^2)} \right|,
\]

where \( r \) is the linear correlation coefficient between the event time and the observed residual, and \( N - 2 \) gives the degrees of freedom (\( N \) is the number of observations). For the BC-DF residuals at COL, FYU, MCK, and SCM, respectively, \( r=0.74, 0.79, 0.68, \) and 0.91. Using the values of \( b, \sigma_b \), and \( N \) given in Table 1, the significance levels at which the null hypothesis is rejected are 0.05% for FYU, 0.01% for MCK, about 7 out of a million for SCM, and less than 1 out of a billion for COL. Note that even though the linear correlation coefficient for the COL observations is relatively small, the increased number of samples from the addition here of the new samples of the early years greatly reduces the significance level (i.e., increases the confidence level) at which the null hypothesis is rejected.

The most important source of errors in the observed temporal changes (Figure 1) is potential systematic event mislocations due to uneven growth of the global station networks used to locate the earthquakes over the decades. We argue, however, that event location errors are unlikely to account for the observed temporal changes. All the events in this study have been relocated using a JHD program written by J. Dewey, which uses a similar set of stations for different events, thus improving relative locations between events and reducing possible systematic biases between events of different periods. P wave arrival time picks were obtained from the following sources for the relocations: the Earthquake Data Report (EDR), the International Seismological Summary (ISS) for events before 1964, the International Seismological Centre (ISC) Bulletin for events after 1964, and the Bureau Central International de Séismologie de France (BCIS) Bulletin for events before 1964. Time picks before 1964 were typed into computer by hand using BCIS, EDR, and ISS catalogs. ISC time picks (after 1964) and EDR time picks after 1990 are available in digital form. We obtained 54 relocated SSI events for COL from the JHD. The differential BC DF time residuals calculated using the JHD locations (Figure 2, top) are compared with those calculated using the EDR locations and the locations from Engdahl et al. [1998] (denoted as EHB), who relocated a subset of ISC events using arrival times of PKP and depth phases, in addition to P and S phases. For events after 1964 (a total of 42 events) for which EDR, JHD, and EHB locations are available, the patterns and the temporal trends of the residuals using the three different sets of locations are remarkably consistent: the slopes of the linear regressions of the residuals are 0.01155(1)\( \pm 0.00148 \) (1\( \sigma \)), 0.00811\( \pm 0.00148 \), and 0.00919\( \pm 0.00137 \) s/yr from the EDR locations, the JHD locations, and the EHB locations, respectively. The consistency and the robustness of the temporal trends using event locations from three independent methods suggest that the time dependence is unlikely to be caused by event mislocations. However, scatter in the BC DF residuals for events in the 1950s, even after the JHD relocations, remains large, which is probably caused by much larger errors in event locations for the early years (Plate 1) due to much fewer reported arrival times (Figure 2, bottom). We note, however, that the station distributions between the 1950s, the 1960s and the 1970s and the 1980s and 1990s used in the JHD overlap with each other in almost all the distance ranges and azimuths (Figure 3); and although the location errors for events in the 1950s are large, the greatest uncertainties are along the direction nearly orthogonal to the azimuth from SSI to COL, minimizing the biases of the mislocations on BC-DF times (Plate 1). To utilize fully these high-quality historical data, it is critical that better event locations be obtained, for example, by including PKP arrival times as in the EHB locations and, possibly, arrival times from regional bulletins.

Independent of the above relocation analysis, a strong argument against the notion of systematic mislocations being the cause of the observed temporal trend was given previously by Song and Richards [1996], who examined the pairs of PKP waves that go through the outer core only, the BC phase and the AB phase. Since the AB path departs from the BC path much more than the separation between the DF path and the BC path, event mislocation has a greater influence on AB BC differential times than on BC-DF differential times. However, we did not observe systematic temporal changes in AB-BC times at COL [Song and Richards, 1996]. Furthermore, the correlation between BC-DF residuals at COL and event magnitudes claimed by Souffrin et al. [1997] was found to be insignificant as discussed extensively in a separate paper by Song and Li (2000). To summarize this section, we conclude that the robustness of the observed trends at four different stations, after our rigorous effort in event relocation, strongly supports the conclusion that the BC-DF times along this pathway have changed over time, presumably caused by some type of inner core rotation.

3. Joint Inversion

The inference of the rotation rate from the observed travel time changes depends on the local lateral velocity changes in the part of the inner core sampled by the paths. We obtained 543 high-quality BC-DF measurements from digital records of SSI earthquakes in the 1990s at ASN stations and COL, which are shown in Figure 4 together with the 68 measurements at COL, FYU, MCK, and SCM from SSI earthquakes before 1990. This large data set confirms significant changes in BC-DF residuals along ray bottoming points in the inner core from east to west (Figure 4), first suggested by Creager [1997]. Imaging the local lateral velocity changes of the inner core, however, can be severely biased by mantle heterogeneity, causing uncertainties in estimating the rotation. However, since the mantle biases do not change as the inner core rotates, they can potentially be separated from time-dependent inner
core structure with dense enough coverage separated in time. Thus we consider an inversion scheme as follows:

\[
\delta v_{\text{obs}} = \delta v(\theta_i, \phi_i, \xi, \tau_i^{IC}, T_i) + \delta v_{\text{mantle}} + \delta v_{\text{mantle}}^{r} + \delta v_{\text{mantle}}^{s},
\]

where \( N \) is the number of observations, \( \delta v_{\text{obs}} \) is the observed BC-DF residuals; and \( \delta v_{\text{mantle}} \) and \( \delta v_{\text{mantle}}^{r} \) are mantle corrections for the path from source \( s \) to receiver (station) \( r \), respectively, both of which are time-independent. The mantle correction terms represent the entire contribution along the path outside the inner core, not necessarily near the station or the source (e.g., in the D' region). The term \( \delta v(\theta_i, \phi_i, \xi, \tau_i^{IC}, T_i) \) represents inner core velocity perturbation averaged along the ray through the inner core with the total accumulated time in the inner core \( \tau_i^{IC} \), the ray angle from the spin axis \( \xi_i \), and the ray bottoming point at latitude \( \theta_i \) and longitude \( \phi_i \) with respect to the mantle reference frame at time \( T_i \). With the averaging, the gradient (i.e., the direction of the steepest change) of \( \delta v \) is perpendicular to the ray direction or along the sampling profile AA' (Figure 4B). Assuming an inner core rotation around the spin axis, the first-order approximation for \( \delta v \) is

\[
\delta v(\theta, \phi, \xi, \tau^{IC}, T) - \delta v_0 + \gamma \frac{\partial v}{\partial \phi} (\phi - \phi_0) - \frac{1}{\gamma} \frac{\partial v}{\partial \phi} \alpha(T - T_0) + \frac{\partial v}{\partial \tau^{IC}} (\tau^{IC} - \tau_0^{IC}) + \Delta v_\xi,
\]

where \( \gamma = 1.10 \) is the ratio of the change in distance \( t \) along AA' to the change in longitude \( \phi \) (thus \( \gamma \) is always \( > 1 \)) and \( \alpha \) is the inner core rotation rate (positive means an eastward rotation). The \( \partial v / \partial \tau^{IC} \) term takes into account a possible change of \( \delta v \) with depth. \( \Delta v_\xi \) is a correction for anisotropy due to slight differences in the ray directions. For a transversely isotropic medium such as the inner core, the velocity pertur-
Plate 1. Error ellipses at 90% confidence level of the SSI event locations from the JHD. The poor locations for the events in the 1950s may be the primary reason for the large scatter in the BC DF times for that period, but note that the major axes of the ellipses are nearly orthogonal to the SSI to COL azimuthal direction.

bution with direction can be expressed as $b \cos^2 \xi + c \cos^4 \xi$, where $b$ and $c$ are constants [Song, 1997]; and $\Delta v_\xi = \frac{b \cos^2 \xi + c \cos^4 \xi}{(b \cos^2 \xi_0 + c \cos^4 \xi_0)}$, where $\xi_0 = 37.2^\circ$ is the average of the ray angles from the spin axis. Here we use $b = 2.56$ and $c = 5.17$ from Song and Richards [1996]; our results do not change if other models (such as that of Creager [1992] or Song and Helmer [1992]) are used. Other parameter values are as follows: $\phi_0 = -78.8^\circ$ is the average of the bottoming point longitudes; $T_{1C}^{0} - 129.3$ s is the average of the travel times through the inner core; and we choose the reference time $T_0 = 1998.0$. Thus the term $-(1/\gamma)\partial v / \partial l \alpha (T - T_0)$ is the travel time change (as a percentage of the total time through the inner core) caused by the shift of the lateral velocity gradient due to the rotation; the effect on travel times is largest for north-south ray paths, given the same amount of the lateral velocity change. Note the $\gamma$ factor in our formulation is the same as the one used by Creager [1997], but the $\alpha$ term in his equation (3) is incorrect: the correct one should be $-\alpha T_i / \gamma$.

The inversion scheme improves upon that of Creager [1997] in that (1) we directly parameterize velocity perturbation $\Delta v$ in terms of the coordinates of the ray in the inner core (instead of the azimuth and distance of the ray); and (2) with 1 order of magnitude more samples distributed more uniformly in space and time, we can invert simultaneously for the inner core lateral gradient and rotation rate, and for the mantle corrections with much finer grids (instead of adjusting COL and surrounding stations to fit a linear lateral gradient, which appears arbitrary).

We perform several least squares (LSQ) inversions, and the results are listed in Table 2. Models 1, 2, and 3 use the whole data set ($N = 611$), and models 4, 5, and 6 use the data in the 1990s only ($N = 376$). To examine mantle biases, we perform inversions with station and source corrections (models 1, and 3), station corrections only (models 2, and 4), and no mantle corrections at all (models 3, and 6). We group the 101 stations and the earthquakes into 21 grids and six grids of $2^\circ \times 2^\circ$ in latitude and longitude, respectively; the gridding is such
Plate 2. Mantle corrections (color) from model 1 at (a) stations in Alaska and (b) the source region in the South Sandwich Islands. The stations (triangles, 101 in total) and the earthquakes (circles, 92 in total) are grouped into 21 and six grids of 2° in latitude and 2° in longitude, respectively. In Plate 2a the contours in thick solid lines are the locations of the Aleutian slab (subducting to the northwest) at 50 km, 100 km, 150 km, and 200 km, respectively. The stations above the slab consistently show larger than average contributions from mantle heterogeneity to the BC-DF times (green to blue). In Plate 2b all the earthquakes have been relocated using the JHD method. The earthquake distributions between different time periods (represented by different colors and symbols) are quite similar.
Plate 3. Maps of the inner core anisotropy inferred from BC-DF travel time residuals at ASN stations from SSI earthquakes in (top) 1990-1992, (middle) 1993-1995, and (bottom) 1996-1998. The observed original residuals (in seconds) are corrected for the \( \delta r_{\text{mantle}} \), \( \delta r_{\text{mantle}} \), \( \partial u/\partial T^{IC} \), and \( \Delta v_k \) terms (equations (1)-(2)) to give the inner core velocity perturbations along the reference ray direction (perpendicular to the sampling profile AA'): \[ \langle \delta r_{\text{obs}} - \delta r_{\text{mantle}} - \delta r_{\text{mantle}} \rangle/T^{IC} - (\partial v/\partial T^{IC})(T^{IC} - T_0) - \Delta v_k = \delta v_0 + \gamma(\partial v/\partial T_0)(\phi - \phi_0) + (1/\gamma)(\partial v/\partial T_0)\alpha(T - T_0). \] The velocity perturbations are plotted at the bottoming points of the DF rays in the inner core (symbols). The circles indicate values smaller than the average of all the perturbations (1990-1998) after the corrections, the crosses indicate values larger than the average. The size of the symbols is proportional to the departures from the average. The colors and the parallel lines represent the linear regressions of the corrected velocity perturbations on the distances along the sampling line AA', calculated independently for each period.
Figure 3. Distance and azimuthal distributions of stations (open circles) used in the JHD relocations of the earthquakes recorded at COL in four periods: 1951-1962 (a total of 17 stations); 1965-1977 (204 stations); 1982-1985 (207 stations); 1990-1995 (272 stations). The radii of dotted circles indicate the epicentral distances from SSI (center) of 30°, 60°, and 90°, respectively; the outer solid circles indicate the epicentral distances of 110°. The azimuths from SSI to stations are plotted clockwise from the top.

that the number of grids is minimum with the given grid size. All the rays to (or from) the same site have the same mantle correction $\delta t_{\text{mantle}}$ (or $\delta t_{\text{mantle}}$). Because the DC component of the velocity perturbation ($\delta v_0$) trades off with the mean of the mantle corrections, we place the conditions that when the corrections are applied, the sum of $\delta t_{\text{mantle}}$ at the 21 grids and the sum of $\delta t_{\text{mantle}}$ at the six grids equal zero in the inversions. This, however, does not affect our inference of the rotation (which has no sensitivity to the DC component) and the inner core lateral velocity gradient.

The station and source corrections for model 1 are shown in Plate 2; the corrections for other models are similar. The station corrections show a remarkable pattern, which appears to correlate with the geometry of the subducting Aleutian slab. The stations above the slab consistently show larger BC-DF residuals than the other stations, and the transition between positive and negative anomalies near the longitude of 145°W coincides with the abrupt change in seismicity and the termination of the Aleutian slab [Page et al., 1991]. The mantle corrections at the source side are much smaller than the corrections at the receiver side.

We examine further the mantle corrections obtained from our joint inversions by comparing them with the predicted perturbations on the BC-DF times for the recent three-dimensional P-velocity tomographic model by van der Hilst et al. [1997] (Figure 5). The model predictions are calculated by first tracing the DF and BC rays through the tomographic model for the 611 earthquake-station pairs, and then averaging the predicted BC-DF perturbations for all the rays reaching the same station grid. We see that our mantle corrections correlate positively with the predictions for the tomographic model. The correlation coefficient is 0.49. Using the t test, the null hypothesis that they do not correlate can be rejected with 97% confidence. The significant positive correlation provides an independent validation of the mantle corrections obtained from our inversions. Note that the predictions for the tomographic model are smaller than our mantle corrections by about a factor of 3. This is not surprising given that
it has been previously suggested from PKP AB-DF differential times that van der Hilst et al.’s [1997] model may underestimate significantly the amplitude of lower mantle heterogeneity [Song and Helmberger, 1997].

The robustness of each term in the inversion can be measured by the corresponding standard deviation using the t test as discussed previously. Of particular importance are two null hypotheses: (1) \( \alpha = 0 \) (no rotation), and (2) \( \alpha < 0 \) (westward rotation). Using estimates of the \( (\partial v/\partial \alpha) \) term in Table 2, the significance levels at which null hypothesis (1) is rejected are 5.7 × 10^{-3}, 3.8 × 10^{-3}, and 1.7 × 10^{-9} for models 1, 2, and 3, respectively, and 5.2 × 10^{-12}, 2.0 × 10^{-12}, and 3.1 × 10^{-14} for models 4, 5, and 6, respectively. For null hypothesis (2), let \( A = (\partial v/\partial \alpha) \) and \( B = (\partial v/\partial \alpha) \); then, the probability \( P(\alpha < 0) \) is \( P(B/A < 0) = P(B > 0)P(A < 0) + P(B < 0)P(A > 0) \). Each probability on the right can be calculated using Table 2 and a one-sided t test. We found that the westward rotation null hypothesis can be rejected at the significance levels of 2.4 × 10^{-7}, 3.7 × 10^{-12}, and 8.6 × 10^{-10} for models 1, 2, and 3, respectively, and 7.5 × 10^{-4}, 1.8 × 10^{-5}, and 1.6 × 10^{-4} for models 4, 5, and 6, respectively.

The above tests indicate, remarkably, that a statistically robust westward inner core rotation is inferred using the ASN observations in the 1990s alone (models 4, 5, and 6), spanning 8 years only. To illustrate this, we divide the data into three separate periods, 1990-1992, 1992-1995, and 1996-1998. The observed travel time residuals are corrected for the perturbations due to mantle heterogeneity and differences in the angles and the depths of the rays through the inner core from model 1. The results are snapshots of the anisotropy of the sampling region beneath Colombia in the inner core at the time periods considered (Plate 3). The lateral velocity gradients obtained separately from the three periods are very similar, confirming that the gradients are robust. More surprisingly, the results clearly show that the image in 1996-1998 (bottom map) has shifted

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**Table 2. Least Squares Inversions for Inner Core Structure and Rotation**

<table>
<thead>
<tr>
<th>Model</th>
<th>df</th>
<th>( \delta v_0, % )</th>
<th>( \delta v_\alpha, %/\alpha^\circ )</th>
<th>( \delta v_\alpha, %/\text{yr} )</th>
<th>( \delta v_{\text{rot}v}, %/\text{s} )</th>
<th>( \alpha, \circ/\text{yr} )</th>
<th>Data Used</th>
<th>Mantle Corrections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>582</td>
<td>2.339</td>
<td>-0.0145</td>
<td>-0.00934</td>
<td>0.0105</td>
<td>0.64</td>
<td>all</td>
<td>station and source</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.013)</td>
<td>(0.0029)</td>
<td>(0.0075)</td>
<td>(0.0018)</td>
<td>(0.14)</td>
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<tr>
<td>2</td>
<td>587</td>
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<td>-0.00953</td>
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<td>0.54</td>
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<tr>
<td></td>
<td></td>
<td>(0.011)</td>
<td>(0.0095)</td>
<td>(0.0076)</td>
<td>(0.0019)</td>
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<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.0016)</td>
<td>(0.00109)</td>
<td>(0.0015)</td>
<td>(0.04)</td>
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<td>(0.017)</td>
<td>(0.0031)</td>
<td>(0.00344)</td>
<td>(0.0018)</td>
<td>(0.86)</td>
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<td>-0.0119</td>
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<td>(0.0029)</td>
<td>(0.00331)</td>
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<td>(0.0017)</td>
<td>(0.00451)</td>
<td>(0.0015)</td>
<td>(0.18)</td>
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The numbers in the parentheses are one standard deviation of the corresponding LSQ solutions. The abbreviation “df” stands for degree of freedom.
to the east relative to the image in 1993-1995 (middle map), which in turn has shifted to the east relative to the image in 1990-1992 (top map).

The LSQ solutions of the inner core rotation rate are simply the \((\partial v/\partial t)\alpha\) term divided by the \(\partial v/\partial t\) term (Table 2). If we use the inner core lateral gradient (\(\partial v/\partial t\)) inferred from model 1 and the temporal changes \((b)\) observed at COI, FYI, MCK, and SCM (Figure 1), the rotation rates \((\alpha = b/[(-1/\gamma)(\partial v/\partial t)])\) are \(0.70\pm0.16^{\circ}/\text{yr}\), \(0.79\pm0.23^{\circ}/\text{yr}\), \(0.87\pm0.25^{\circ}/\text{yr}\), and \(1.12\pm0.27^{\circ}/\text{yr}\), respectively. If only the data in the 1990s are used, the rotation rates and their uncertainties are much larger. If no mantle corrections are applied, the inferred inner core lateral velocity gradients increase and the inferred rotation rates decrease.

We assume that the axis of the inner core anisotropy is aligned with the spin axis in the above analyses; what give rise to the observed temporal changes in the travel times are the lateral velocity changes. Previous studies have indicated that the anisotropy axis may be tilted by as much as \(11^{\circ}\) from the spin axis [Su and Dziewonski, 1995; McSweeney et al., 1997; Song and Richards, 1996], although such a tilt may not be resolvable at present [Souriau et al., 1997]. If the local anisotropy axis of this patch of the inner core (not necessarily the anisotropy axis averaged over the whole inner core) is indeed tilted from the spin axis (our assumed axis of the inner core rotation), the change of the orientation of the axis as the inner core rotates would contribute to the change of the velocity perturbations sampled by a fixed path (i.e., the \(\Delta v_\xi\) term in equation (2)) is time-dependent), as originally proposed by Song and Richards [1996]. In fact, the new observations obtained at COL going further back in time agree well with Song and Richards's [1996] model (Figure 2).

If we assume that the anisotropy axis is tilted by \(10^{\circ}\), but allow its longitude to vary, the LSQ solution for equations (1)-(2) that include both station and source correction terms to fit the whole data set puts the longitude of the anisotropy axis at \(99\pm22^{\circ}\) in the beginning of 1998 with a rotation rate of \(0.31\pm0.04^{\circ}/\text{yr}\). The solutions for \(\partial v/\partial t\) (which is \(-0.0150\pm0.0016^{\circ}/\text{yr}\) and the station and source corrections are all very close to the corresponding solutions without a tilt of the anisotropy axis (model 1), which is reasonable since all of our rays travel in very similar directions through the inner core (with \(1\sigma\) less than \(1.2^{\circ}\)). Thus our inferred lateral velocity gradients are not biased by a possible, but unknown, tilt of the anisotropy axis. On the other hand, our inferred rotation rate varies with tilt of the anisotropy axis. For a small rotation the time-dependent \(\Delta v_\xi\) term, to the first order, can be approximated (Figure 6) by \((2bR_s^2+4cR_s^2)\alpha(T-T_0)(-R_xA_0-R_yA_0)\), where \((R_xA_0,A_0)\) are the Cartesian coordinates of the ray direction and the anisotropy axis (at time \(T_0\), respectively. For all of the paths, the \(\Delta v_\xi\) terms are all around \(0.0136\alpha(T-T_0)^{\%}\), com-

**Figure 5.** Solid circles connected by the solid line are the mantle corrections at the 21 station grids of \(2^\circ\times2^\circ\) determined in this study by the joint inversion with the mantle corrections involving station terms only (model 2). The station grids are sorted in ascending order from west to east and from south to north (Plate 2a). Open circles connected by the dashed line are the predicted perturbations on the BC-DF times for the LV-fit tomographic model by van der Hilst et al. [1997]. The predictions, with the small (-0.04 s) mean removed, are amplified by a factor of 3 in the plot. The mantle corrections determined in this study correlate positively with the predictions for the tomographic model.

**Figure 6.** Schematic diagram for calculating the changes of inner core velocity perturbations (\(\Delta v_\xi\) of equation (2)) sampled by a fixed path (R) due to a small rotation of the anisotropy axis (from A0 to A) in the case when the anisotropy axis is tilted from the assumed north-south rotation axis (N) \(\Delta v_\xi = (h\sin^2\xi + r\cos^2\xi) - (h\sin^2\xi + r\cos^2\xi_0)\), where \(\xi\) (or \(\xi_0\)) is the angle between A (or A0) and R; thus \(\cos\xi = -R\cdot A\) and \(\cos\xi_0 = R\cdot A_0\). For a small tilt and a small rotation (i.e., the angle between A or A0 and N and the angle between A and A0 in radians are \(\ll 1\)), \(\Delta v_\xi\) can be approximated by a linear relation with time (given in the text).
parable to the change due to the local lateral velocity gradient $-1/(T - T_0)$. Thus if we consider the shifts of both the lateral velocity gradient $(\partial \vec{v}/\partial T = -0.0150\%)$ and the anisotropy axis (with the assumed tilt of $10^\circ$), the rotation rates inferred from the temporal changes observed at COL, FYU, MCK, and SCM are $0.34 \pm 0.05^\circ/yr$, $0.38 \pm 0.09^\circ/yr$, $0.42 \pm 0.10^\circ/yr$, and $0.54 \pm 0.09^\circ/yr$, respectively.

4. Concluding Remarks

We have established that the temporal changes of the differential BC-DF travel times at ASN and COL stations are robust, which can be interpreted by a differential inner core rotation that shifts the lateral velocity changes present at this part of the inner core and, possibly, the orientation of the anisotropy axis (if it is tilted from the inner core rotation axis). The lateral velocity gradient is found to be robust and very significant, but some $50\%$ of the lateral changes in original residuals (Figure 4) can be explained by mantle heterogeneities. Our estimates of the rotation rate obtained above that include the whole data set and station and source corrections fall within a tight range from $0.31^\circ/yr$ to $1.12^\circ/yr$, even when a tilt of up to $10^\circ$ is considered: the average is $0.82 \pm 0.19^\circ/yr$ when no tilt of the anisotropy axis is assumed and about half of that when a $10^\circ$ tilt is assumed. The average of these two end-member models is $0.61 \pm 0.26^\circ/yr$.

So far, we have considered only the simplest mode of inner-core rotation, a steady rotation around the Earth’s spin axis. While the dynamo simulation by Glatzmaier and Roberts [1996] almost always suggests a super-rotation of the inner core of a few degrees per year, the inner core in Kung and Bloxham’s [1997] dynamo rotates either eastward or westward at smaller rates, even though the two simulations produce very similar magnetic fields outside the core. In addition to the electromagnetic torque, the inner core may also be influenced by gravitational coupling between the mantle and small topography of the inner core induced by mantle heterogeneity [Buffett, 1996]. The inner core is expected to oscillate [Gubbins, 1981; Aurnou and Olson, 2000], at the time scale of years to decades [Aurnou and Olson, 2000] depending on relative strength of the two torques. Although the proof of such an oscillation will be challenging because of the precision and data sampling required, our conclusion that the differential inner core rotation is apparently resolvable with as short as eight years’ data raises the hope that the proof may indeed be available.

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References


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